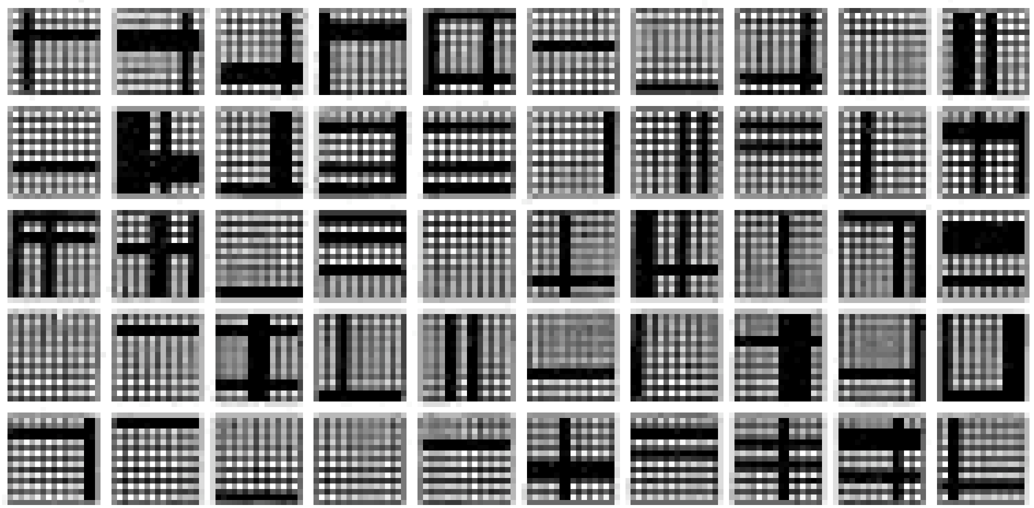
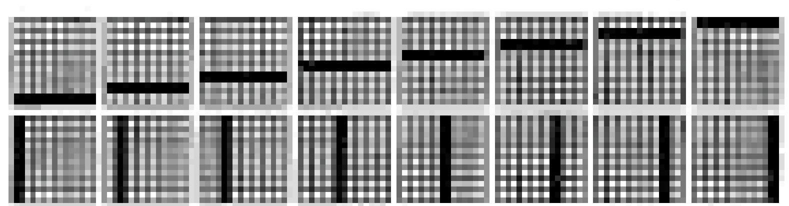
Why *linear* generative models?

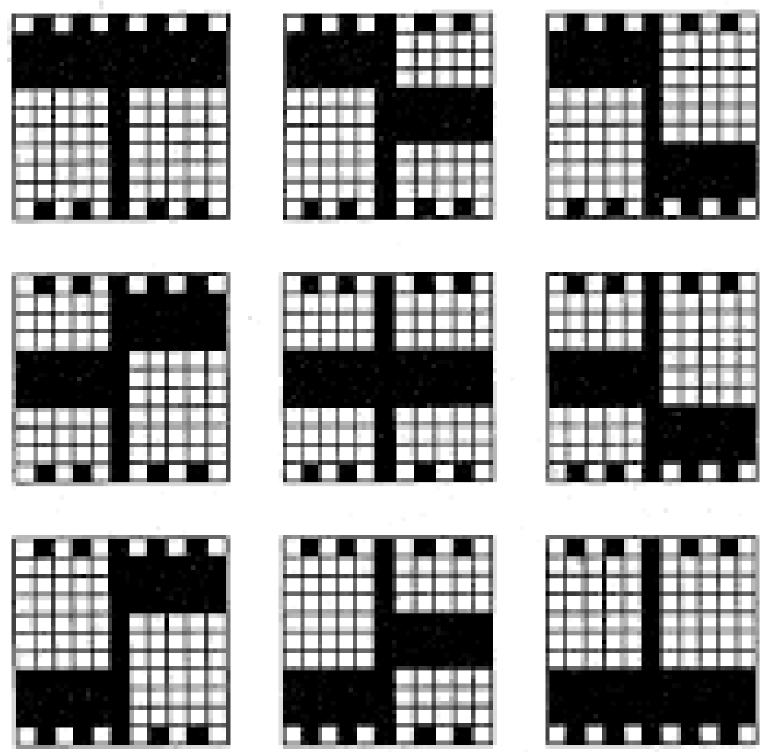
$$I(x, y) = \sum_i a_i\, \phi_i(x, y) + \nu(x, y)$$

The bars problem



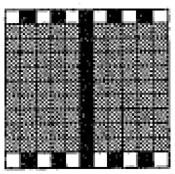


Eric Saund's *write-white-and-black* model

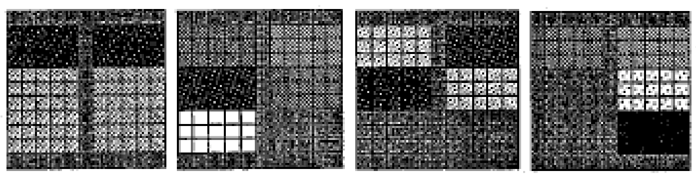


PCA

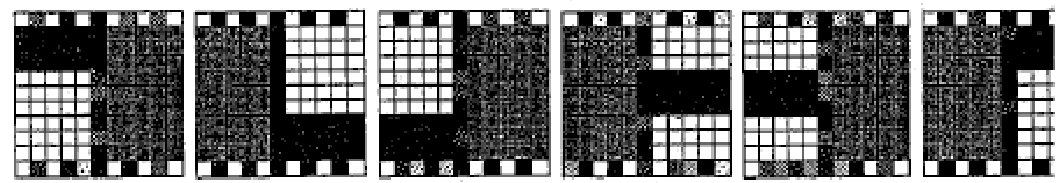
write-white-and-black



a



b



b

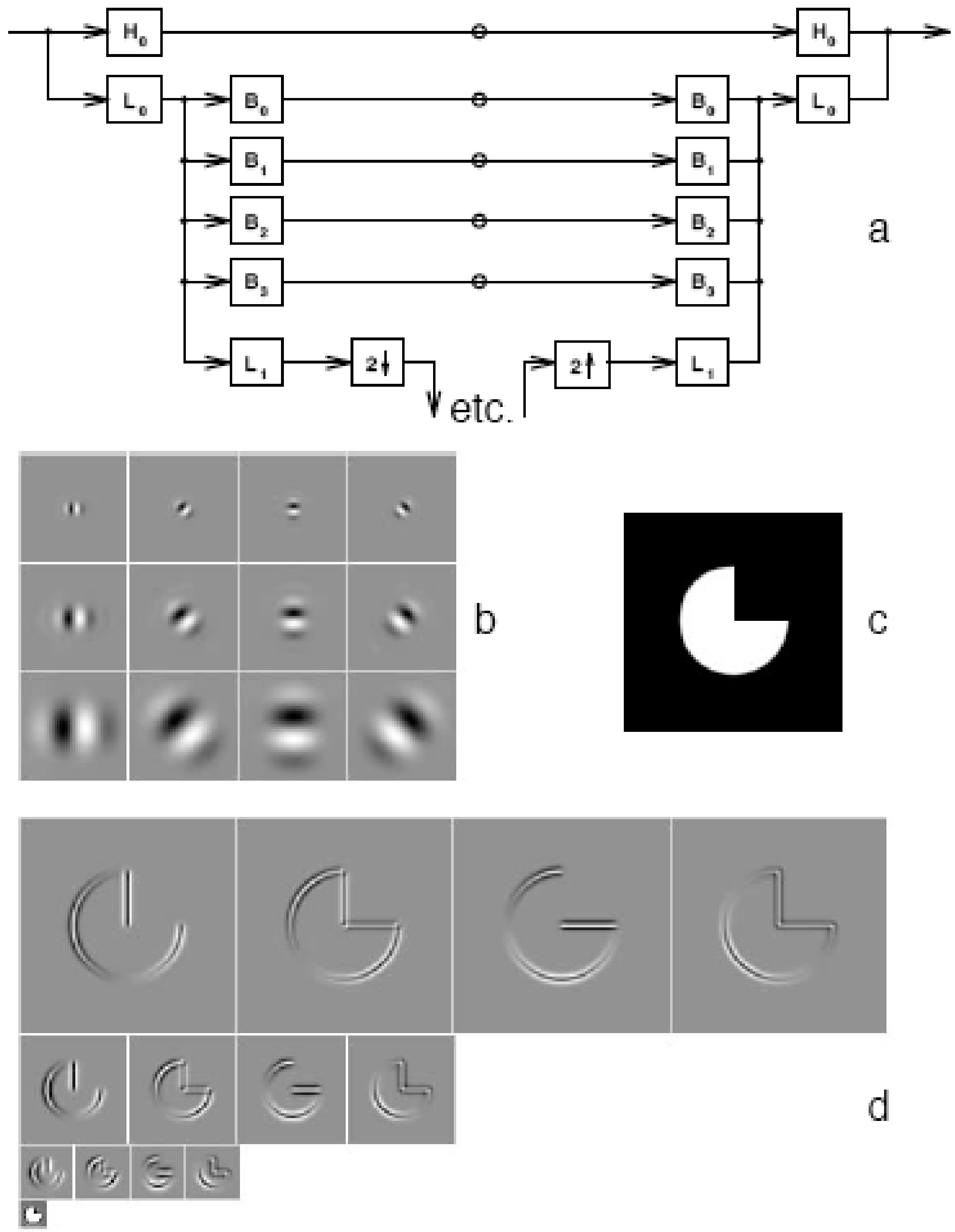
c

1.0	0.0	0.0	0.0	0.0	1.0
1.0	0.0	0.0	1.0	0.0	0.0
1.0	1.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	1.0
0.0	0.0	0.0	1.0	1.0	0.0
0.0	1.0	0.0	0.0	1.0	0.0
0.0	0.0	1.0	0.0	0.0	1.0
0.0	0.0	1.0	1.0	0.0	0.0
0.0	1.0	1.0	0.0	0.0	0.0

1.49	0.0	-0.0	-0.0
.373	.539	-1.2	-.59
.373	.873	-.75	1.17
.373	.374	1.36	-.31
-.75	.912	.156	-0.9
-.75	.747	.606	.862
.373	-1.3	.597	-.27
-.75	-.75	-.61	-.86
-.75	-.91	-.16	0.9

c

Heeger & Bergen's texture model



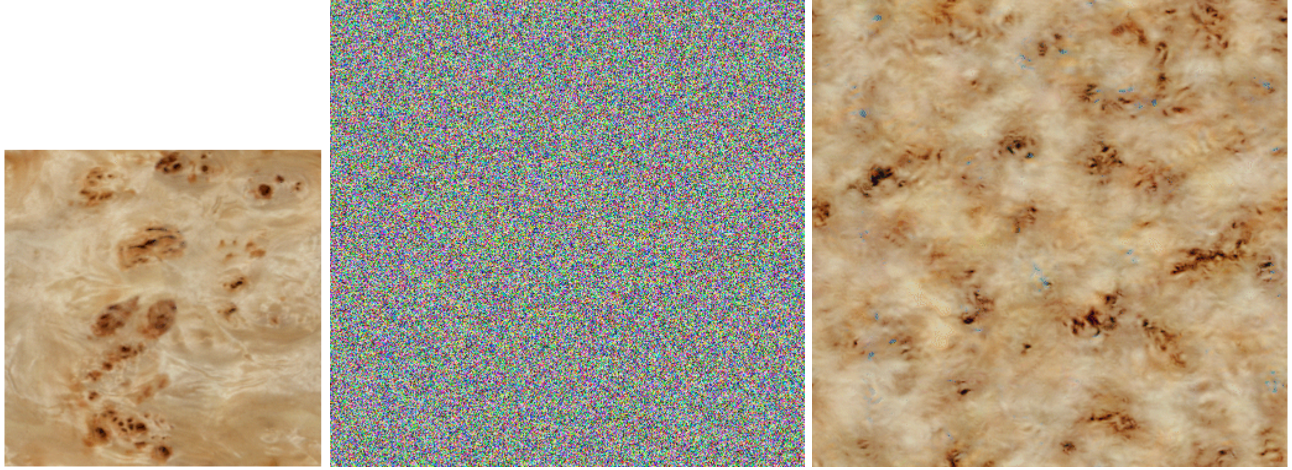


Figure 2: (Left) Input digitized sample texture: burled mappa wood. (Middle) Input noise. (Right) Output synthetic texture that matches the appearance of the digitized sample. Note that the synthesized texture is larger than the digitized sample; our approach allows generation of as much texture as desired. In addition, the synthetic textures tile seamlessly.

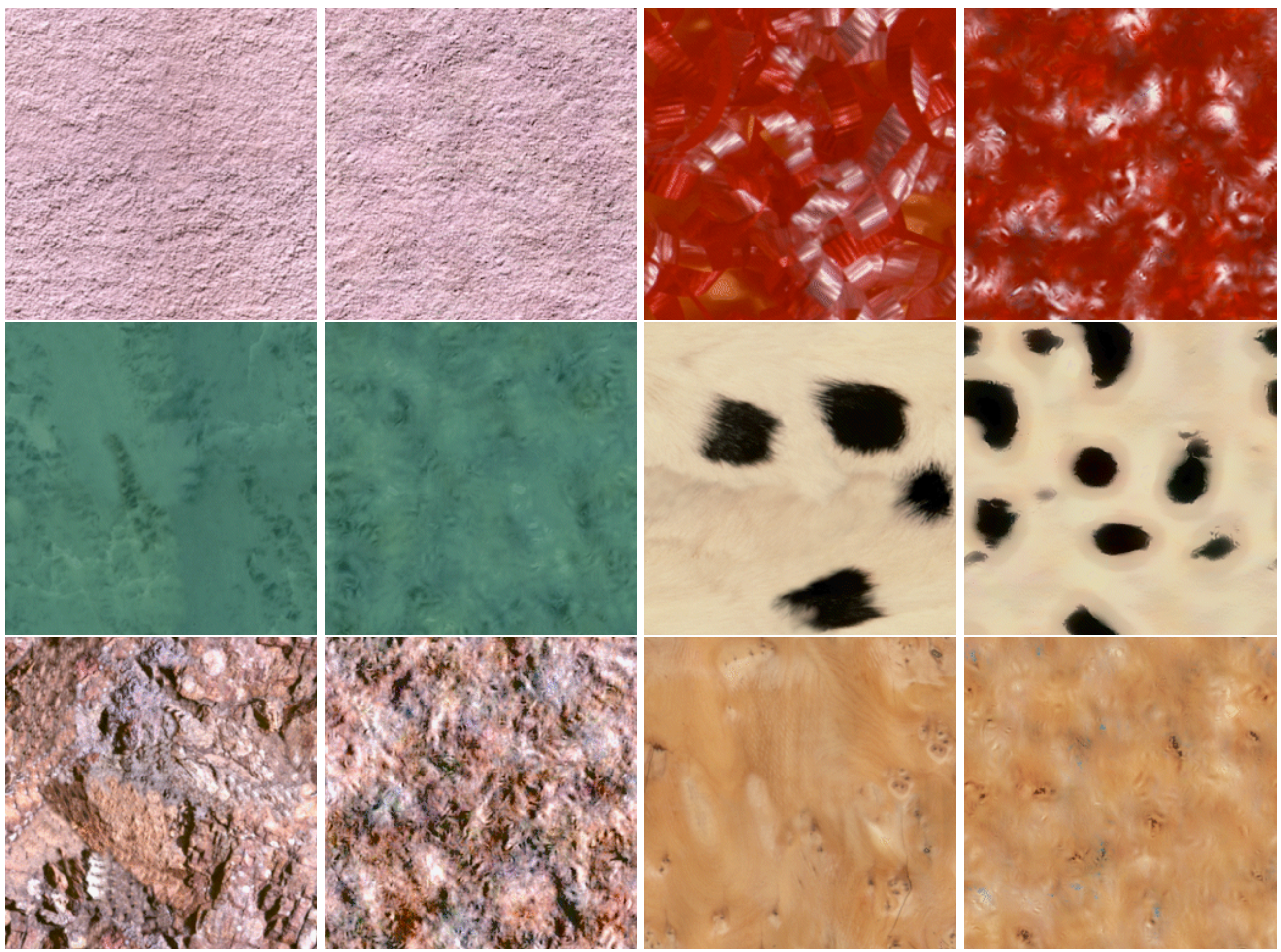
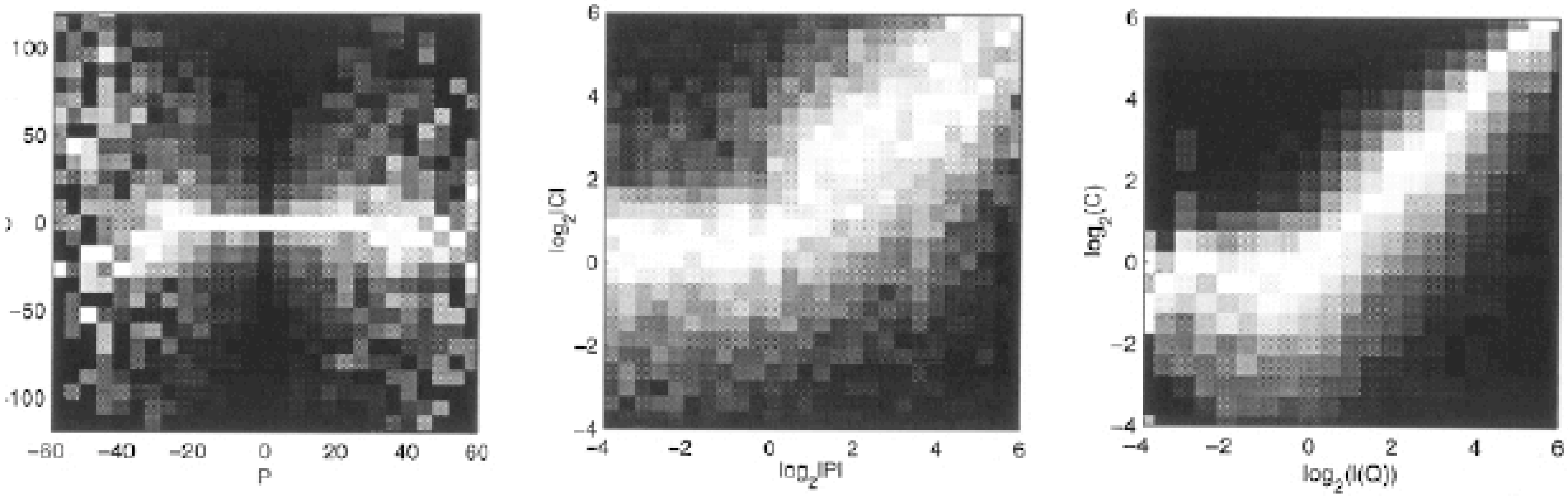
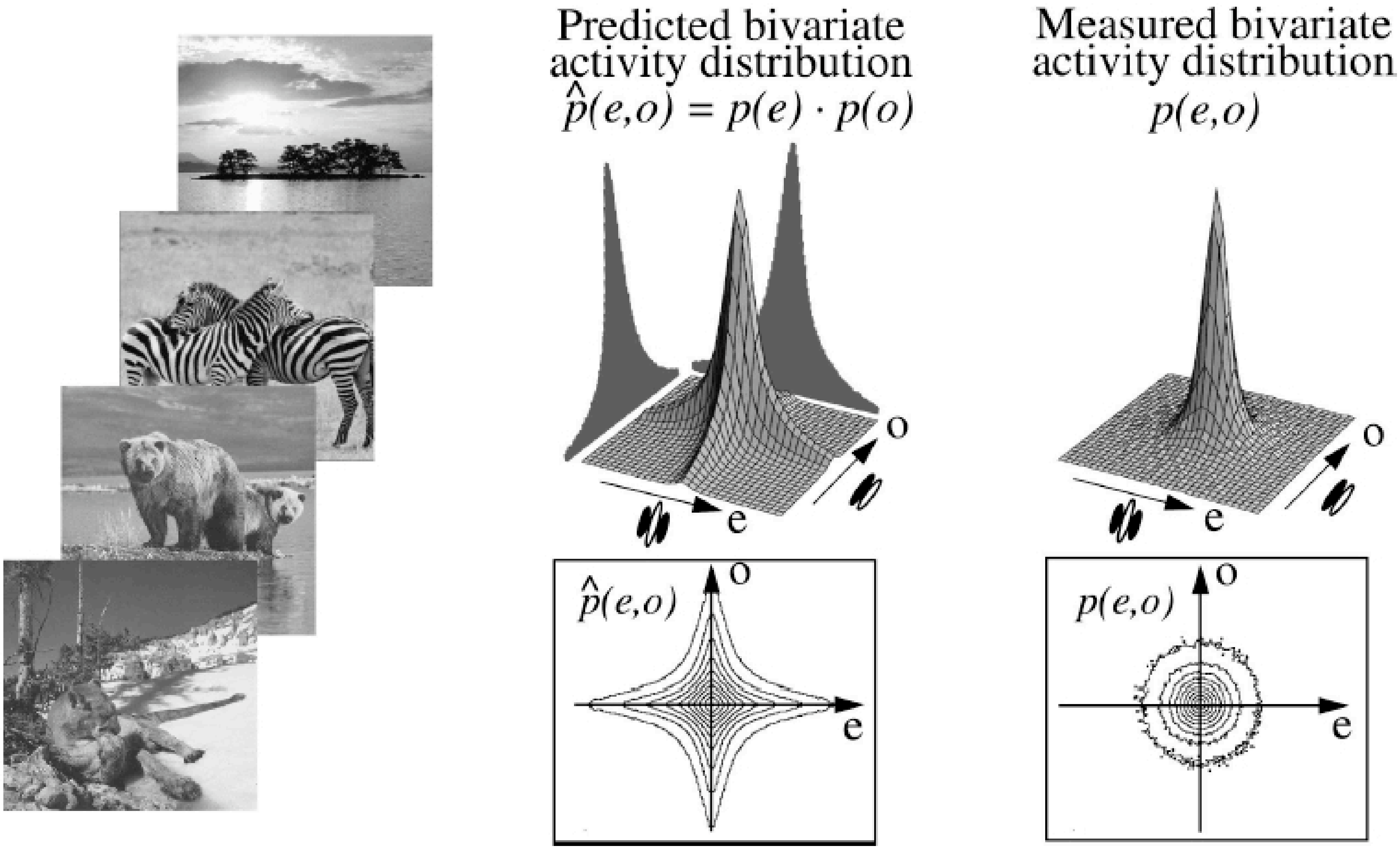


Figure 3: In each pair left image is original and right image is synthetic: stucco, iridescent ribbon, green marble, panda fur, slag stone, figured yew wood.

Statistical dependencies among coefficients

Buccigrossi & Simoncelli (1997)



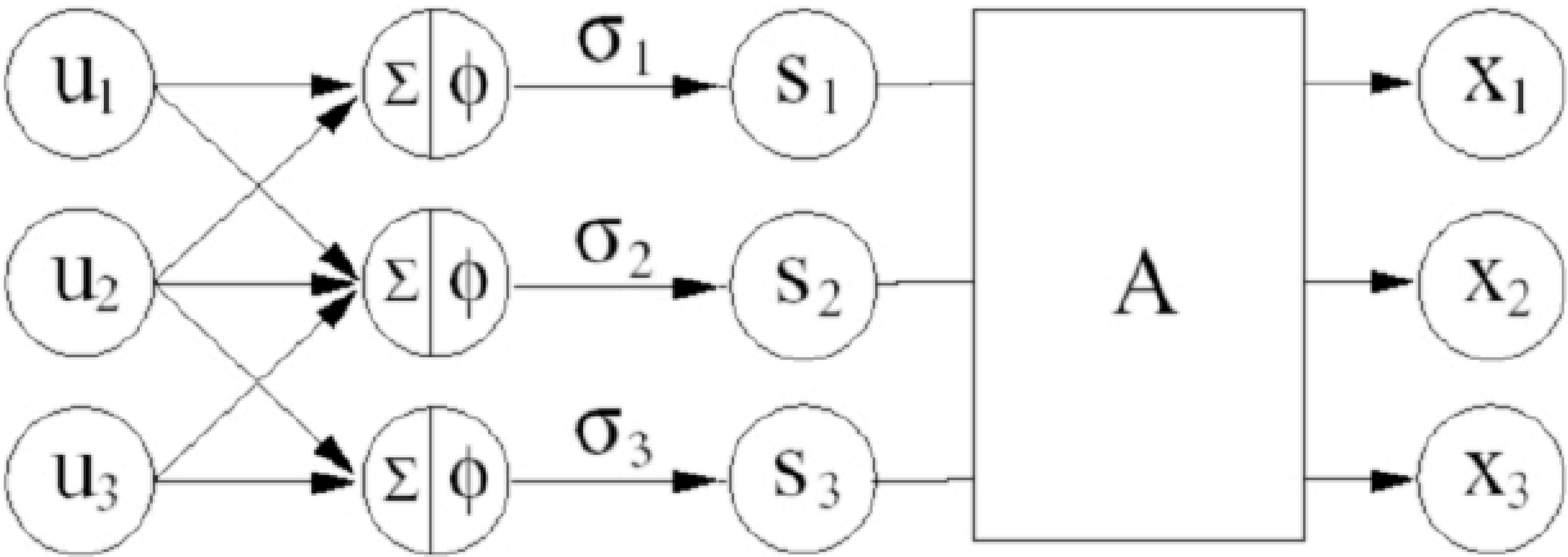

Predicted bivariate activity distribution
$\hat{p}(e,o) = p(e) \cdot p(o)$
o
e
$\hat{p}(e,o)$
o
e
Measured bivariate activity distribution
$p(e,o)$
o
e
$p(e,o)$
o
e

Hierarchical models for capturing dependencies among sparse components

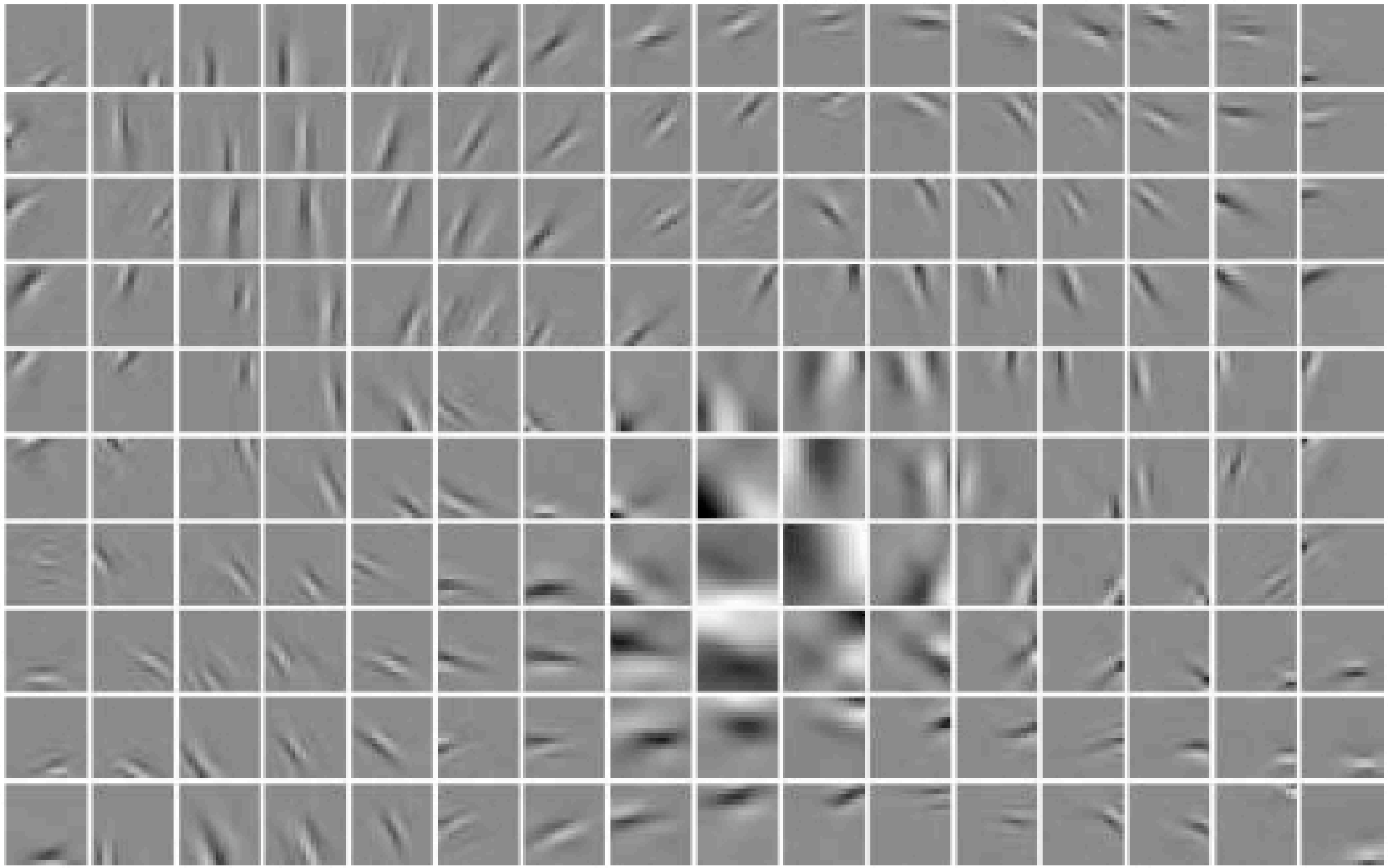
$$
\begin{aligned}
a_i &= \overbrace{\sigma_i}^{\text{power-law}} \times \overbrace{z_i}^{\text{Gaussian}} \\
\sigma_i &= f\Big(\sum_j \Psi_{ij}\, b_j\Big)
\end{aligned}
$$

Wainwright & Simoncelli (2002)
Hyvarinen & Hoyer (2002)
Karklin & Lewicki (2003)
Schwartz & Sejnowski (2004)
Osindero & Hinton (2005)

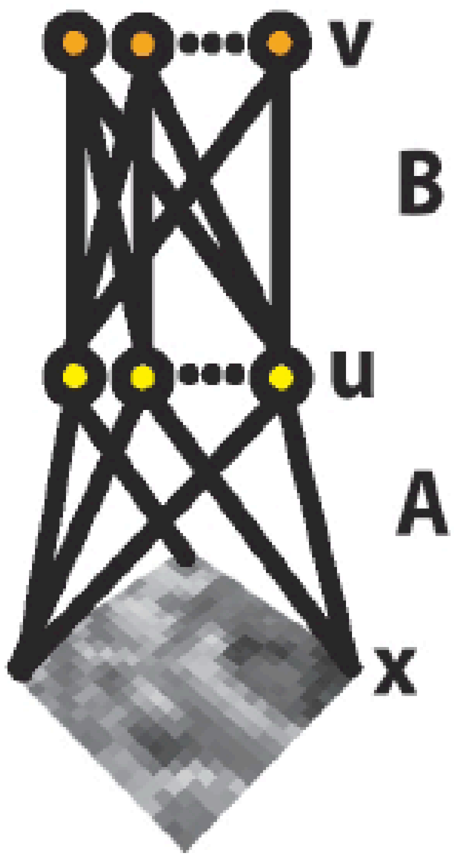
'Topographic ICA' - Hyvarinen & Hoyer (2002)



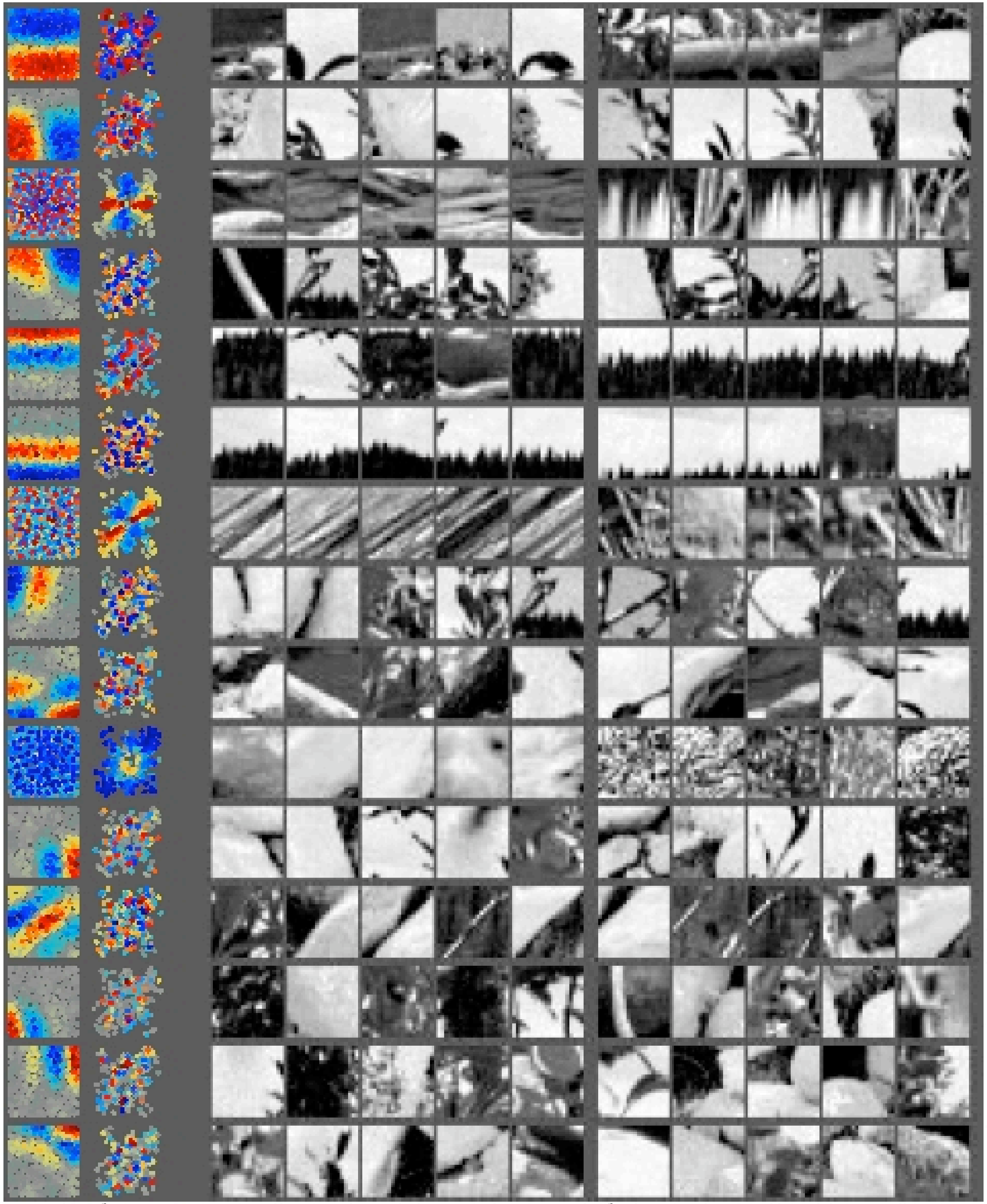
'Topographic ICA' - Hyvarinen & Hoyer (2002)



Learning the neighborhoods - Karklin & Lewicki (2003)



$$\mathbf{x} = \mathbf{A}\,\mathbf{u}$$
$$u_i = \sigma_i\, z_i$$
$$\sigma_i = e^{\sum_j B_{ij} v_j}$$



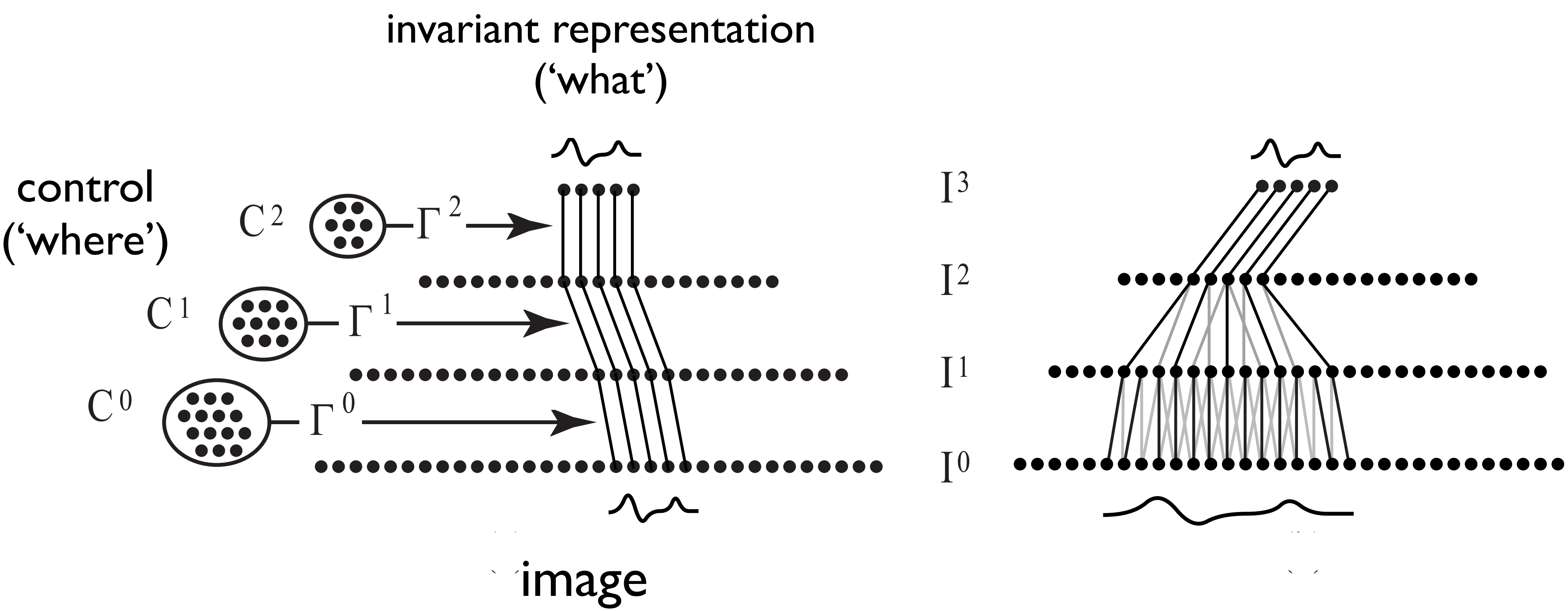
Bilinear models for learning invariant representations

$$\mathbf{z} = \sum_{ij} \mathbf{w}_{ij} \underbrace{x_i}_{\text{`what'}} \underbrace{y_j}_{\text{`where'}}$$

- Tenenbaum & Freeman (2000) - SVD
- Grimes & Rao (2005) - sparse coding

Dynamic routing circuits for forming invariant representations

(Olshausen, Anderson & Van Essen 1993)



Generative model for transformation and shape

$$
\begin{aligned}
I^0(x) &= \sum_{x'} T(x,x')\, I^1(x') \\
T(x,x') &= \sum_j c_j\, \Psi_j(x,x') \\
I^1(x) &= \sum_i a_i\, \phi_i(x)
\end{aligned}
$$

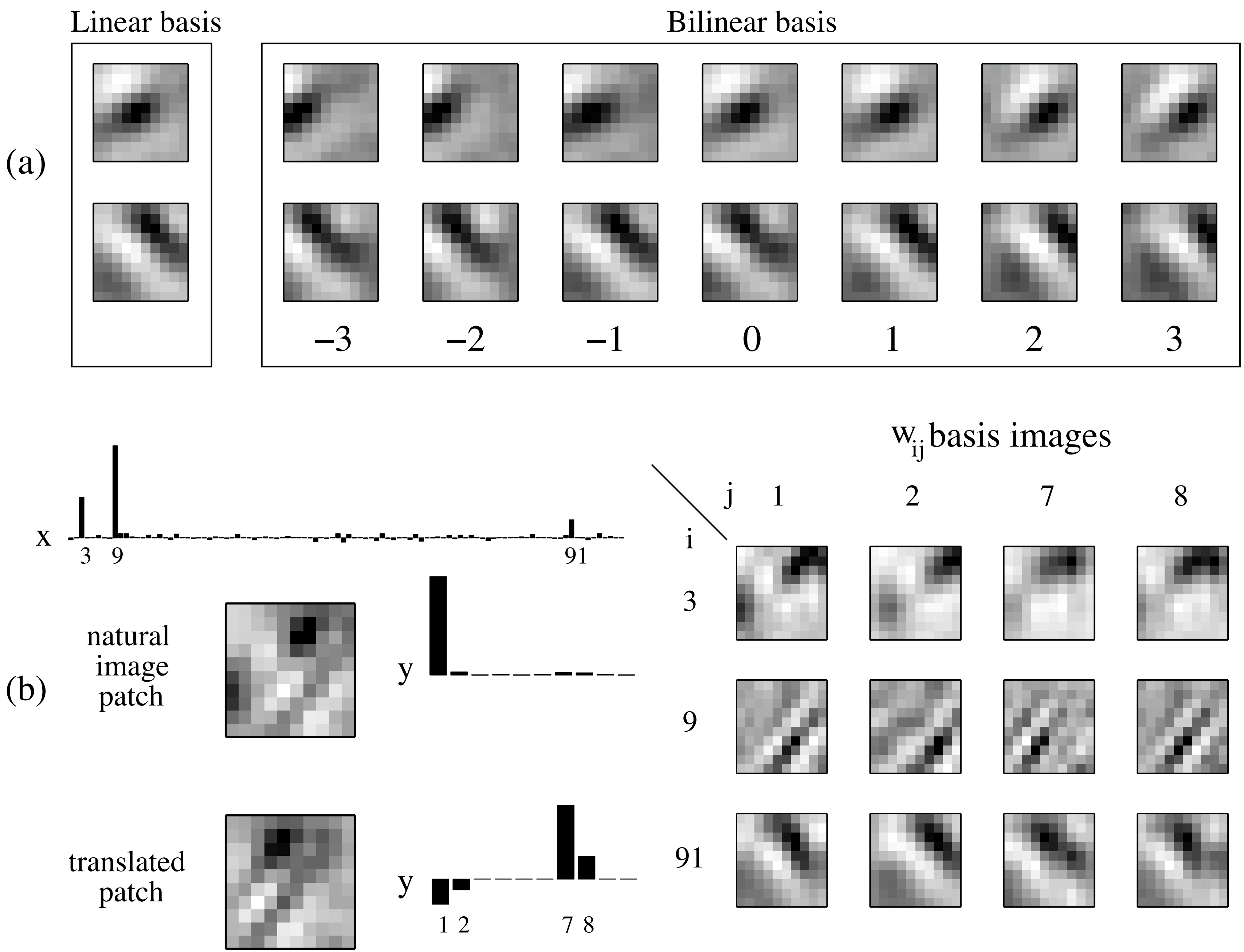
Define:

$$\Gamma_{ij}(x) = \sum_{x'} \Psi_j(x,x')\, \phi_i(x')$$

Then:

$$I^0(x) = \sum_{ij} \Gamma_{ij}(x)\, a_i\, c_j$$

(from Grimes & Rao)



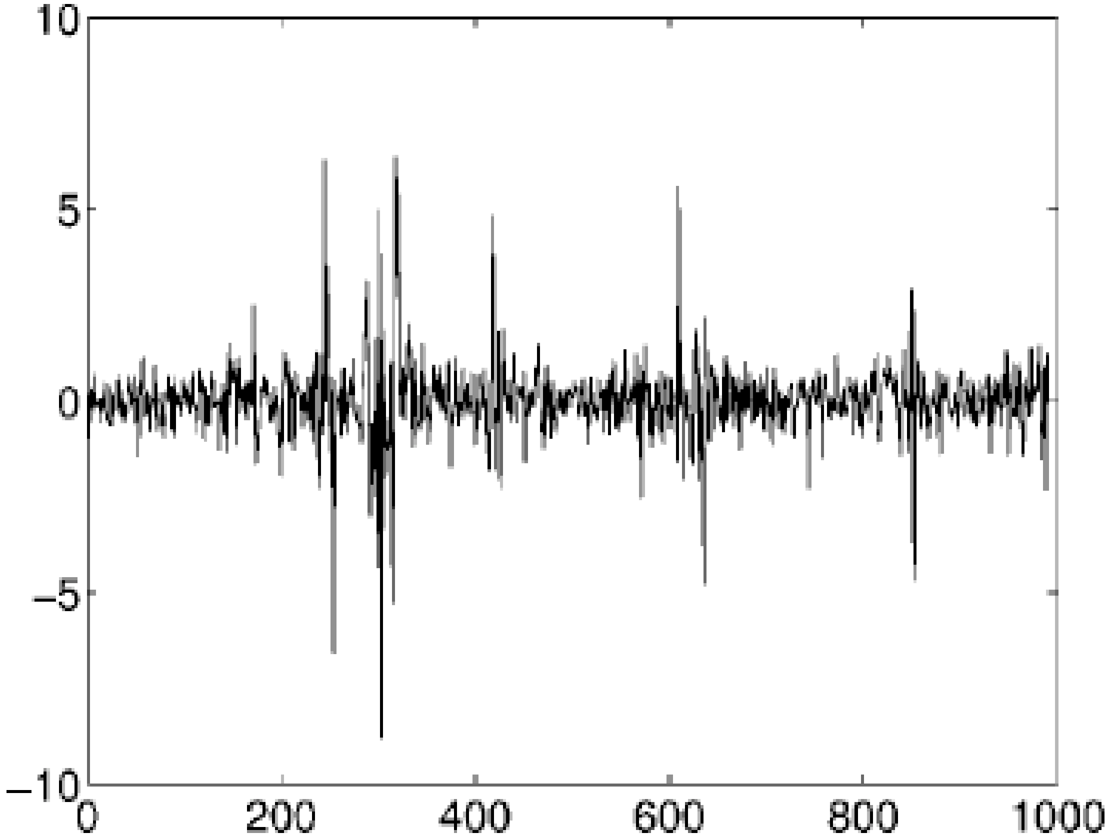
Slow feature analysis

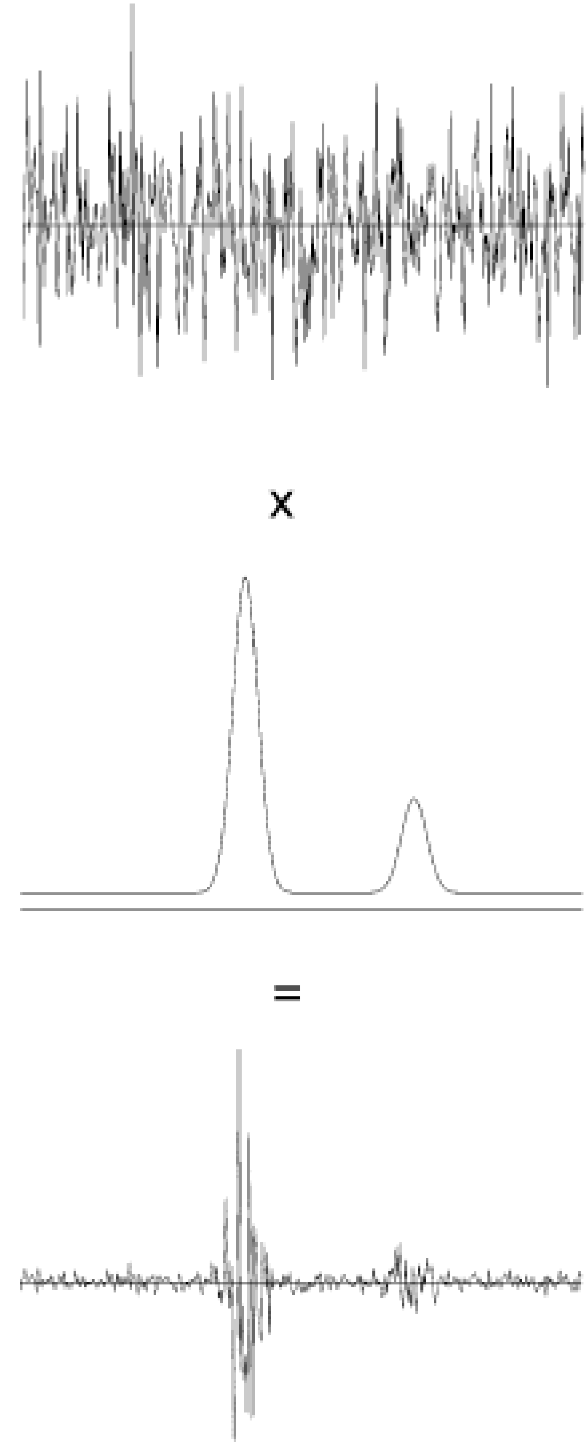
Foldiak (1991), Wiskott (2002)

Learn invariant causes by imposing slowness over time.

Bubbles

Hyvarinen et al. (2003) JOSA *20*



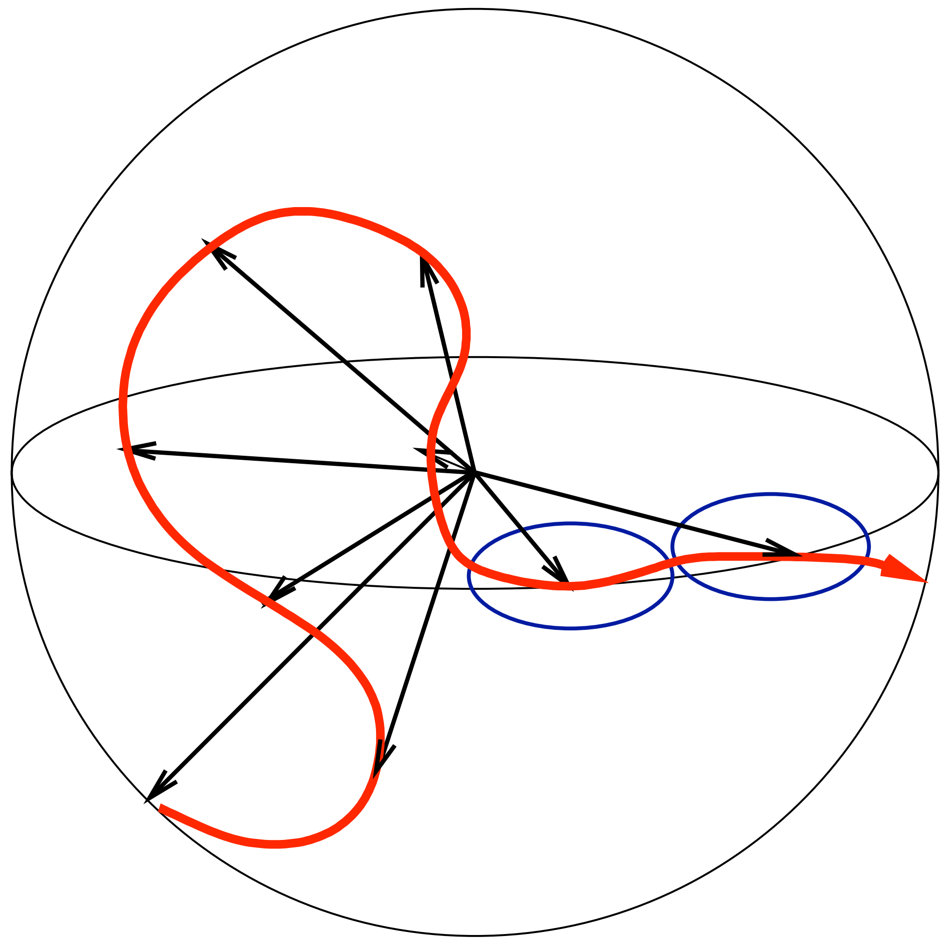


Generative model

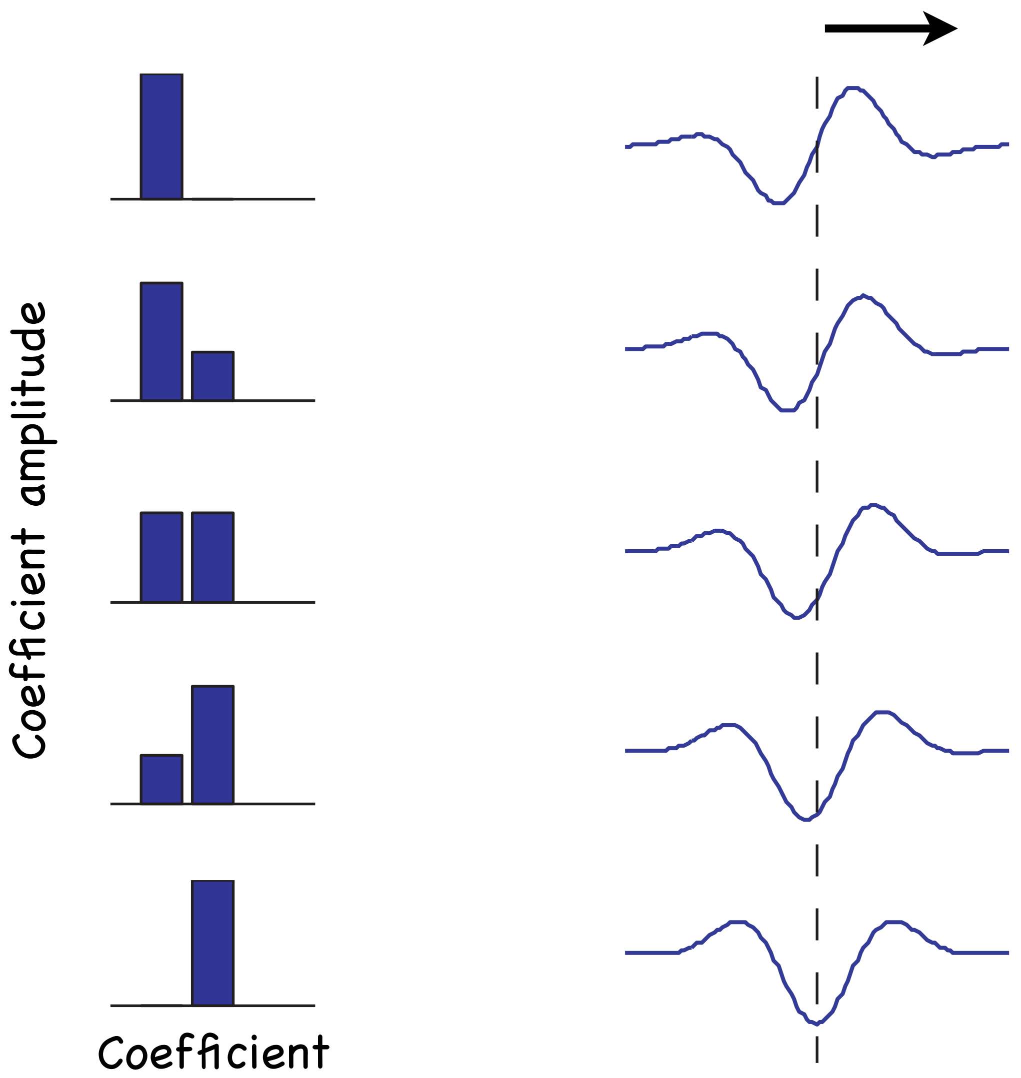
$$I(x,y,t) = \sum_i a_i(t)\,\phi_i(x,y)$$

$$a_i(t) = \underbrace{\sigma_i(t)}_{\text{slow,sparse}}\ \underbrace{z_i(t)}_{\text{fast}}$$

Invariance manifolds



Translation via phase shifting in complex wavelets



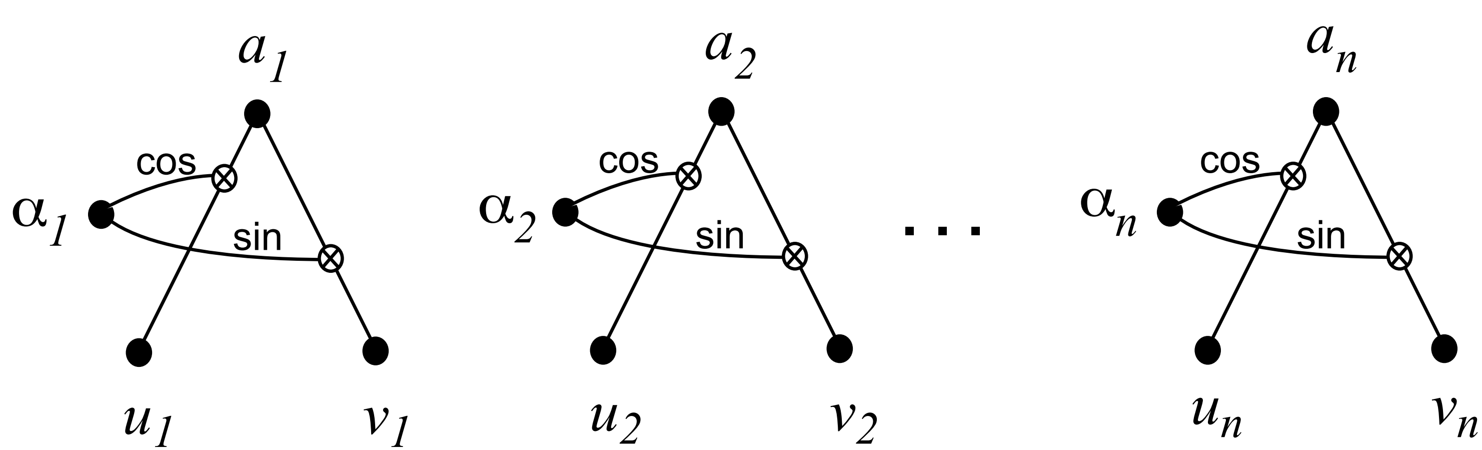
Complex basis function model

$$I(x,y) = \sum_i \Re\{z_i\,\phi_i(x,y)\}$$

$$z_i = a_i\,e^{j\,\alpha_i}$$

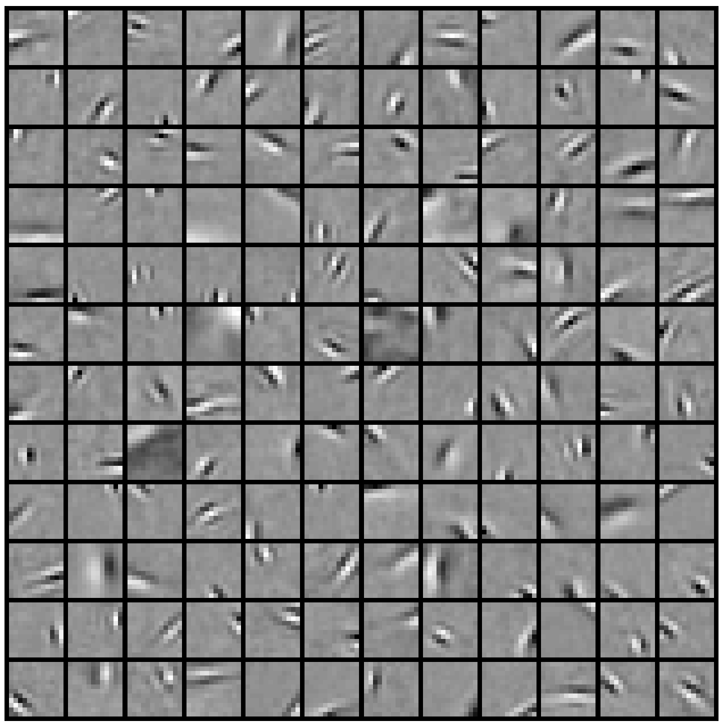
$$\phi_i(x,y) = \phi_i^R(x,y) + j\,\phi_i^I(x,y)$$

$$\boxed{I(x,y) = \sum_i a_i\left[\cos\,\alpha_i\;\phi_i^R(x,y) + \sin\,\alpha_i\;\phi_i^I(x,y)\right]}$$

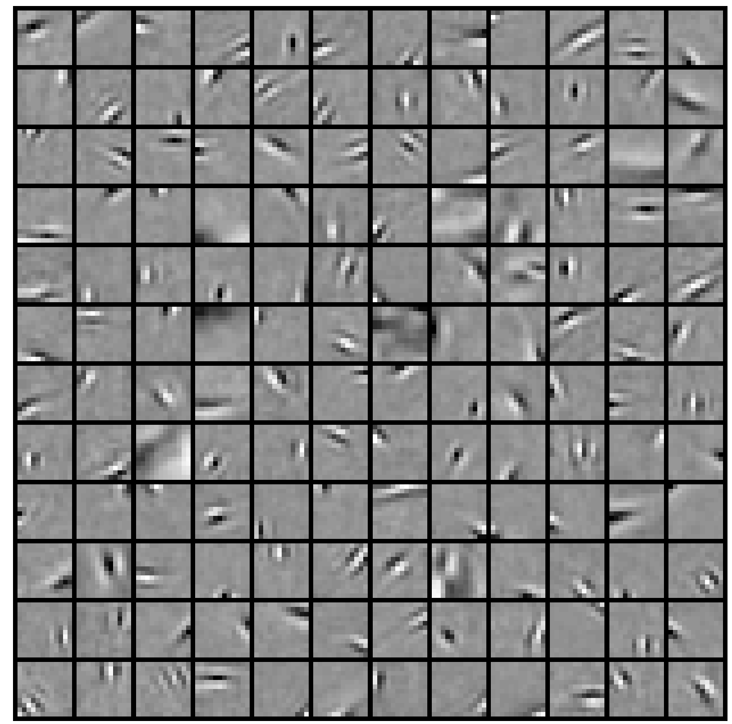


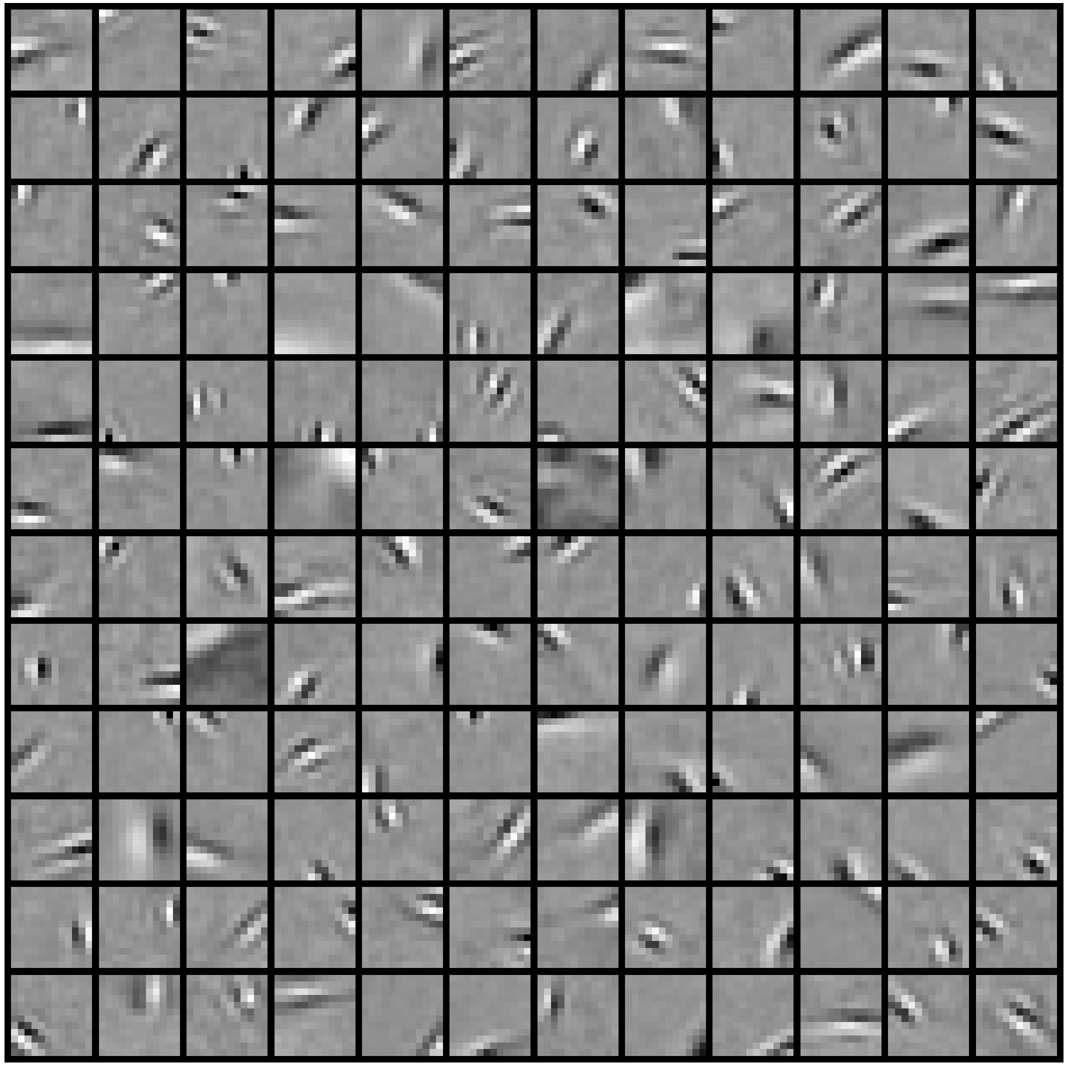
Learned complex basis functions

real



imag





Texture synthesis from complex wavelets

(Portilla & Simoncelli)